

Exploring Angle Pairs

Objective To identify special angle pairs and use their relationships to find angle measures



It might help if you make a sketch of the pieces and cut them out.



Getting Ready!

The five game pieces at the right form a square to fit back in the box. Two of the shapes are already in place. Where do the remaining pieces go? How do you know? Make a sketch of the completed puzzle.



In this lesson, you will learn how to describe different kinds of angle pairs.

Essential Understanding Special angle pairs can help you identify geometric relationships. You can use these angle pairs to find angle measures.



Lesson Vocabulary

- adjacent angles
- vertical angles
- complementary angles
- supplementary angles
- linear pair
- angle bisector

Take Note

Key Concept Types of Angle Pairs

Definition

Adjacent angles are two coplanar angles with a common side, a common vertex, and no common interior points.

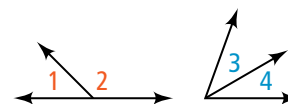
Vertical angles are two angles whose sides are opposite rays.

Complementary angles are two angles whose measures have a sum of 90. Each angle is called the *complement* of the other.

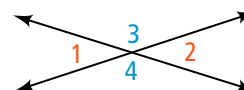
Supplementary angles are two angles whose measures have a sum of 180. Each angle is called the *supplement* of the other.

Example

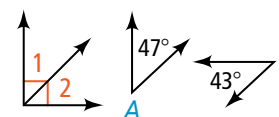
$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$



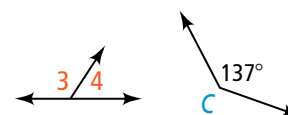
$\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$



$\angle 1$ and $\angle 2$, $\angle A$ and $\angle B$



$\angle 3$ and $\angle 4$, $\angle B$ and $\angle C$



Plan

What should you look for in the diagram?

For part (A), check whether the angle pair matches every part of the definition of adjacent angles.

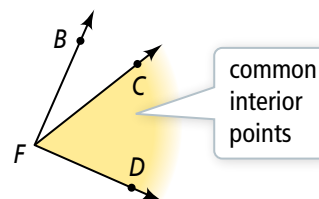


Problem 1 Identifying Angle Pairs

Use the diagram at the right. Is the statement true? Explain.

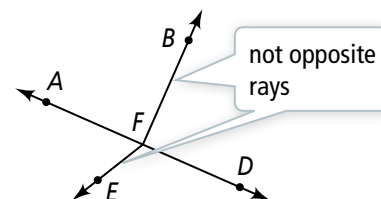
A $\angle BFD$ and $\angle CFD$ are adjacent angles.

No. They have a common side (\overrightarrow{FD}) and a common vertex (F), but they also have common interior points. So $\angle BFD$ and $\angle CFD$ are not adjacent.



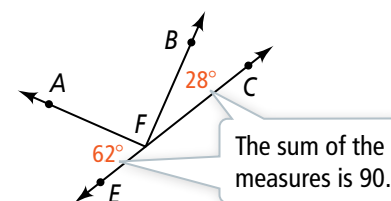
B $\angle AFB$ and $\angle EFD$ are vertical angles.

No. \overrightarrow{FA} and \overrightarrow{FD} are opposite rays, but \overrightarrow{FE} and \overrightarrow{FB} are not. So $\angle AFB$ and $\angle EFD$ are not vertical angles.



C $\angle AFE$ and $\angle BFC$ are complementary.

Yes. $m\angle AFE + m\angle BFC = 62 + 28 = 90$. The sum of the angle measures is 90, so $\angle AFE$ and $\angle BFC$ are complementary.



Got It? 1. Use the diagram in Problem 1. Is the statement true? Explain.

- $\angle AFE$ and $\angle CFD$ are vertical angles.
- $\angle BFC$ and $\angle DFE$ are supplementary.
- $\angle BFD$ and $\angle AFB$ are adjacent angles.

Take note

Concept Summary Finding Information From a Diagram

There are some relationships you can assume to be true from a diagram that has no marks or measures. There are other relationships you cannot assume directly. For example, you *can* conclude the following from an unmarked diagram.

- Angles are adjacent.
- Angles are adjacent and supplementary.
- Angles are vertical angles.

You *cannot* conclude the following from an unmarked diagram.

- Angles or segments are congruent.
- An angle is a right angle.
- Angles are complementary.

Think

How can you get information from a diagram?

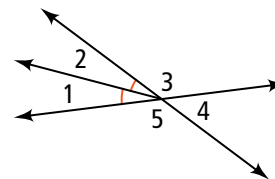
Look for relationships between angles. For example, look for congruent angles and adjacent angles.



Problem 2 Making Conclusions From a Diagram

What can you conclude from the information in the diagram?

- $\angle 1 \cong \angle 2$ by the markings.
- $\angle 3$ and $\angle 5$ are vertical angles.
- $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, $\angle 4$ and $\angle 5$, and $\angle 5$ and $\angle 1$ are adjacent angles.
- $\angle 3$ and $\angle 4$, and $\angle 4$ and $\angle 5$ are adjacent supplementary angles.
So, $m\angle 3 + m\angle 4 = 180$ and $m\angle 4 + m\angle 5 = 180$ by the definition of supplementary angles.



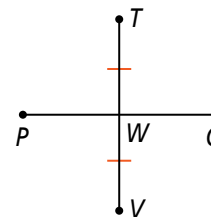
Got It? 2. Can you make each conclusion from the information in the diagram? Explain.

a. $\overline{TW} \cong \overline{WV}$

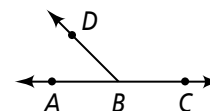
b. $\overline{PW} \cong \overline{WQ}$

c. $\angle TWQ$ is a right angle.

d. \overline{TV} bisects \overline{PQ} .



A **linear pair** is a pair of adjacent angles whose noncommon sides are opposite rays. The angles of a linear pair form a straight angle.



take note

Postulate 1-9 Linear Pair Postulate

If two angles form a linear pair, then they are supplementary.



Problem 3 Finding Missing Angle Measures

Algebra $\angle KPL$ and $\angle JPL$ are a linear pair, $m\angle KPL = 2x + 24$, and $m\angle JPL = 4x + 36$. What are the measures of $\angle KPL$ and $\angle JPL$?

Know

$\angle KPL$ and $\angle JPL$ are supplementary.

Need

$m\angle KPL$ and $m\angle JPL$

Plan

Draw a diagram. Use the definition of supplementary angles to write and solve an equation.

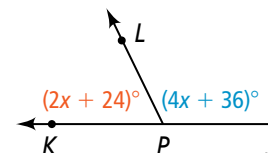
- Step 1** $m\angle KPL + m\angle JPL = 180$ Def. of supplementary angles
 $(2x + 24) + (4x + 36) = 180$ Substitute.
 $6x + 60 = 180$ Combine like terms.
 $6x = 120$ Subtract 60 from each side.
 $x = 20$ Divide each side by 6.

Step 2 Evaluate the original expressions for $x = 20$.

$m\angle KPL = 2x + 24 = 2 \cdot 20 + 24 = 40 + 24 = 64$

$m\angle JPL = 4x + 36 = 4 \cdot 20 + 36 = 80 + 36 = 116$

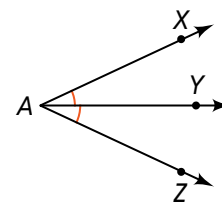
Substitute 20 for x .





- Got It? 3. a. Reasoning** How can you check your results in Problem 3?
- b. $\angle ADB$ and $\angle BDC$ are a linear pair. $m\angle ADB = 3x + 14$ and $m\angle BDC = 5x - 2$. What are $m\angle ADB$ and $m\angle BDC$?

An **angle bisector** is a ray that divides an angle into two congruent angles. Its endpoint is at the angle vertex. Within the ray, a segment with the same endpoint is also an angle bisector. The ray or segment bisects the angle. In the diagram, \overrightarrow{AY} is the angle bisector of $\angle XAZ$, so $\angle XAY \cong \angle YAZ$.



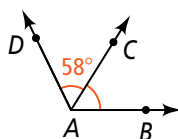
Problem 4 Using an Angle Bisector to Find Angle Measures

Multiple Choice \overrightarrow{AC} bisects $\angle DAB$. If $m\angle DAC = 58$, what is $m\angle DAB$?

- (A) 29 (B) 58 (C) 87 (D) 116

Plan

Draw a diagram to help you visualize what you are given and what you need to find.



Draw a diagram.

$$\begin{aligned} m\angle CAB &= m\angle DAC \\ &= 58 \end{aligned}$$

Definition of angle bisector

Substitute.

$$\begin{aligned} m\angle DAB &= m\angle CAB + m\angle DAC \\ &= 58 + 58 \\ &= 116 \end{aligned}$$

Angle Addition Postulate

Substitute.

Simplify.

The measure of $\angle DAB$ is 116. The correct choice is D.



- Got It? 4.** \overrightarrow{KM} bisects $\angle JKL$. If $m\angle JKL = 72$, what is $m\angle JKM$?

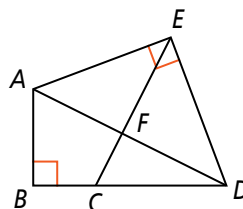


Lesson Check

Do you know HOW?

Name a pair of the following types of angle pairs.

- vertical angles
- complementary angles
- linear pair



- \overrightarrow{PB} bisects $\angle RPT$ so that $m\angle RPB = x + 2$ and $m\angle TPB = 2x - 6$. What is $m\angle RPT$?

Do you UNDERSTAND?



MATHEMATICAL PRACTICES

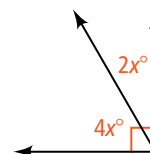


- 5. Vocabulary** How does the term *linear pair* describe how the angle pair looks?



- 6. Error Analysis** Your friend calculated the value of x below. What is her error?

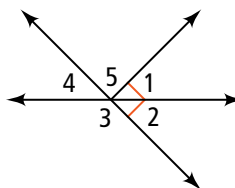
$$\begin{aligned} 4x + 2x &= 180 \\ 6x &= 180 \\ x &= 30 \end{aligned}$$



A Practice

Use the diagram at the right. Is each statement true? Explain.

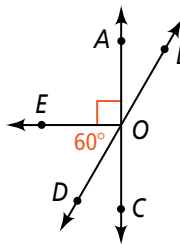
7. $\angle 1$ and $\angle 5$ are adjacent angles.
8. $\angle 3$ and $\angle 5$ are vertical angles.
9. $\angle 3$ and $\angle 4$ are complementary.
10. $\angle 1$ and $\angle 2$ are supplementary.



See Problem 1.

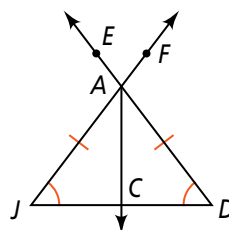
Name an angle or angles in the diagram described by each of the following.

11. supplementary to $\angle AOD$
12. adjacent and congruent to $\angle AOE$
13. supplementary to $\angle EOA$
14. complementary to $\angle EOD$
15. a pair of vertical angles



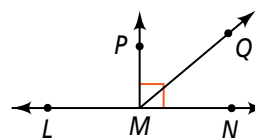
For Exercises 16–23, can you make each conclusion from the information in the diagram? Explain.

16. $\angle J \cong \angle D$
17. $\angle JAC \cong \angle DAC$
18. $m\angle JCA = m\angle DCA$
19. $m\angle JCA + m\angle ACD = 180$
20. $\overline{AJ} \cong \overline{AD}$
21. C is the midpoint of \overline{JD} .
22. $\angle JAE$ and $\angle EAF$ are adjacent and supplementary.
23. $\angle EAF$ and $\angle JAD$ are vertical angles.



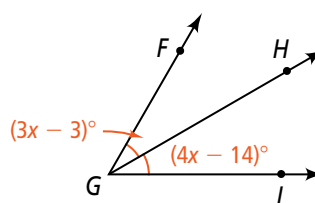
See Problem 2.

24. Name two pairs of angles that form a linear pair in the diagram at the right.
25. $\angle EFG$ and $\angle GFH$ are a linear pair, $m\angle EFG = 2n + 21$, and $m\angle GFH = 4n + 15$. What are $m\angle EFG$ and $m\angle GFH$?



See Problem 3.

26. **Algebra** In the diagram, \overrightarrow{GH} bisects $\angle FGI$.
 - a. Solve for x and find $m\angle FGH$.
 - b. Find $m\angle HGI$.
 - c. Find $m\angle FGI$.



See Problem 4.

B Apply

Algebra \overrightarrow{BD} bisects $\angle ABC$. Solve for x and find $m\angle ABC$.

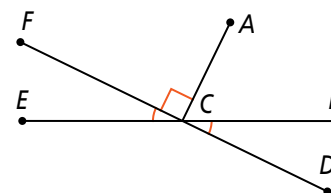
27. $m\angle ABD = 5x$, $m\angle DBC = 3x + 10$
28. $m\angle ABC = 4x - 12$, $m\angle ABD = 24$
29. $m\angle ABD = 4x - 16$, $m\angle CBD = 2x + 6$
30. $m\angle ABD = 3x + 20$, $m\angle CBD = 6x - 16$

Algebra Find the measure of each angle in the angle pair described.

- © 31. **Think About a Plan** The measure of one angle is twice the measure of its supplement.
- How many angles are there? What is their relationship?
 - How can you use algebra, such as using the variable x , to help you?
32. The measure of one angle is 20 less than the measure of its complement.

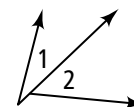
In the diagram at the right, $m\angle ACB = 65$. Find each of the following.

33. $m\angle ACD$
34. $m\angle BCD$
35. $m\angle ECD$
36. $m\angle ACE$



37. **Algebra** $\angle RQS$ and $\angle TQS$ are a linear pair where $m\angle RQS = 2x + 4$ and $m\angle TQS = 6x + 20$.
- a. Solve for x .
 - b. Find $m\angle RQS$ and $m\angle TQS$.
 - c. Show how you can check your answer.

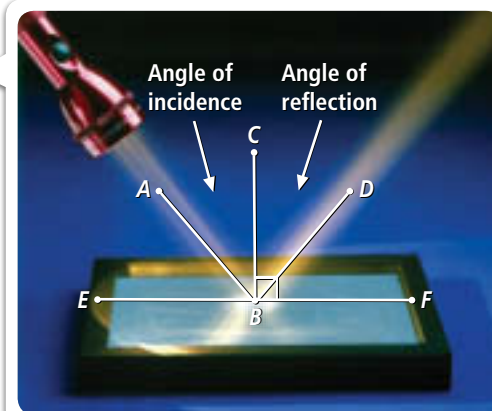
- © 38. **Writing** In the diagram at the right, are $\angle 1$ and $\angle 2$ adjacent? Justify your reasoning.



- © 39. **Reasoning** When \overrightarrow{BX} bisects $\angle ABC$, $\angle ABX \cong \angle CBX$. One student claims there is always a related equation $m\angle ABX = \frac{1}{2} m\angle ABC$. Another student claims the related equation is $2m\angle ABX = m\angle ABC$. Who is correct? Explain.

- STEM** 40. **Optics** A beam of light and a mirror can be used to study the behavior of light. Light that strikes the mirror is reflected so that the angle of reflection and the angle of incidence are congruent. In the diagram, $\angle ABC$ has a measure of 41.
- a. Name the angle of reflection and find its measure.
 - b. Find $m\angle ABD$.
 - c. Find $m\angle ABE$ and $m\angle DBF$.

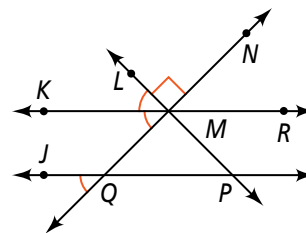
- © 41. **Reasoning** Describe all situations where vertical angles are also supplementary.





Name all of the angle(s) in the diagram described by the following.

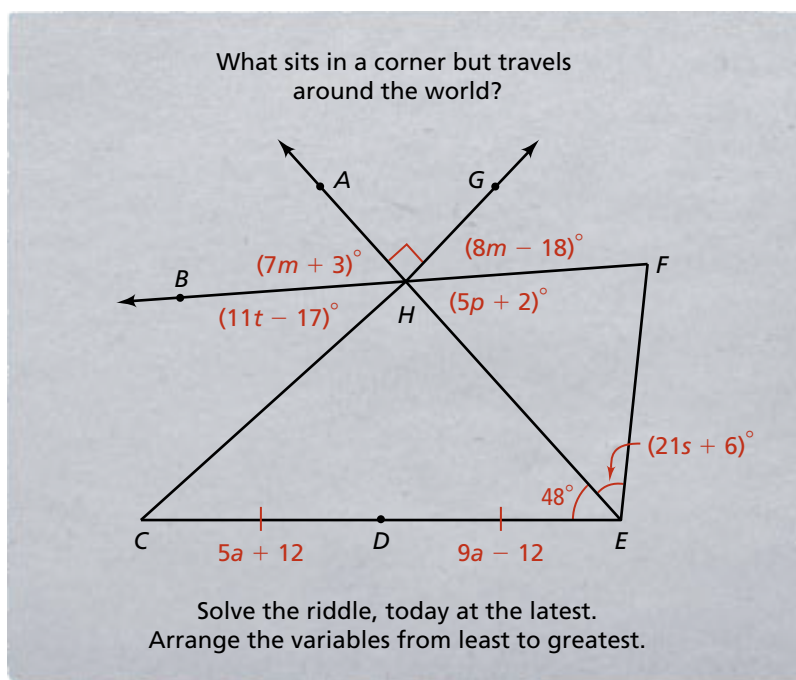
42. supplementary to $\angle JQM$ 43. adjacent and congruent to $\angle KMQ$
 44. a linear pair with $\angle LMQ$ 45. complementary to $\angle NMR$
 46. **Coordinate Geometry** The x - and y -axes of the coordinate plane form four right angles. The interior of each of the right angles is a quadrant of the coordinate plane. What is the equation for the line that contains the angle bisector of Quadrants I and III?
 47. \overrightarrow{XC} bisects $\angle AXB$, \overrightarrow{XD} bisects $\angle AXC$, \overrightarrow{XE} bisects $\angle AXD$, \overrightarrow{XF} bisects $\angle EXD$, \overrightarrow{XG} bisects $\angle EXF$, and \overrightarrow{XH} bisects $\angle DXB$. If $m\angle DXC = 16$, find $m\angle GXH$.



Apply What You've Learned



Look back at the information on page 3 about the riddle Cameron found in an antique store. The page from the old riddle book is shown again below.



- Name a pair of adjacent complementary angles in the diagram. Explain how you know they are complementary.
- Name a pair of nonadjacent complementary angles in the diagram.
- In the Apply What You've Learned sections in Lessons 1-3 and 1-4, you found the values of the variables a and s . Which variable's value can you find next? Find the value of this variable.